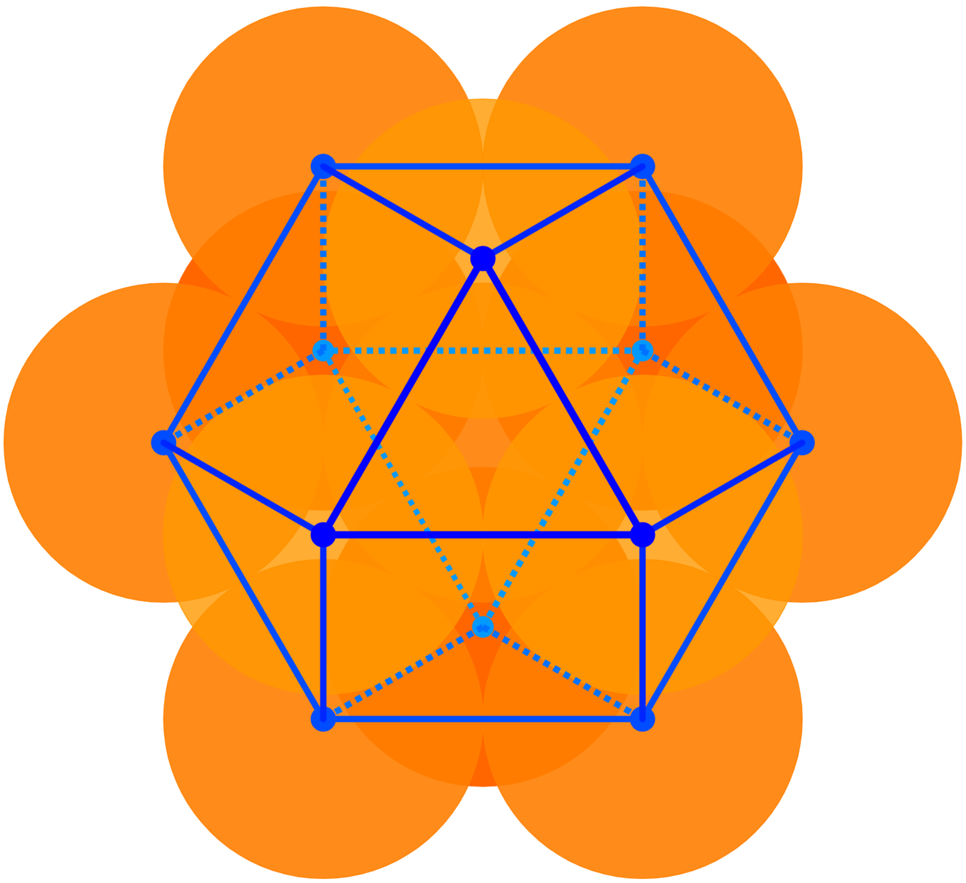
Orange Stack: Solution

There are three layers and a central sphere is chosen from the middle one; it has three neighboring spheres in both the top and bottom layers, and six neighboring spheres in the middle layer, for a total of twelve neighbors.



Counting, we find 12 vertices, 24 edges, and 14 faces. Specifically, 8 triangular and 6 square faces - the quadrilateral faces may not look like squares in the projection because in 3D they are sloped. This is a **cuboctahedron**.

This illustrates the **kissing number** (the most unit spheres which fit around a central one) in 3D is 12. In 2D the kissing number is 6, corresponding to a hexagon, which also extends to a circle packing. (This is why honeycombs use hexagons!) In 4D the kissing number is 24, corresponding to a 24-cell.

The centers of the spheres in the three layers, projected onto a single plane, form three copies of a triangular lattice. Call the three copies a, b, c. Any infinite sequence of letters a, b, c (with no consecutive ones repeated) gives a way to layer spheres atop each other filling all of 3D space. The densest **sphere packings** in 3D all arise this way. The two most common are $\cdots abcabc \cdots$ and $\cdots ababab \cdots$, called FCC (face-centered cubic) and HFP (hexagonal close packing) respectively.

The densest *regular* sphere packings are known in dimensions ≤ 8. In 4D, the 24-cell extends to such a packing - the centers form the Hurwitz quaternions.

In 2016 Maryna Viazovska showed that the E_8 lattice is the densest sphere packing in eight dimensions, and later (with collaboration) that the Leech lattice is densest in twenty-four dimensions, regular or irregular. Her proof uses theta functions and is considered surprisingly simple compared to the proof for 3D (the next highest dimension the densest packing was proven).