Creating a Hubble diagram for a galaxy cluster using Hubble's methods

Austin Aiken
Department of Physics, University of Nebraska Omaha, Omaha, NE 68128
(Dated: May 22, 2024)

We present a comprehensive analysis of the relationship between galaxy redshifts and relative distances, resulting in a construction of a Hubble diagram. We use the magnitude and spectrum line data from the Sloan Digital Sky Survey (SDSS). By combining redshift measurements from spectroscopy and distance estimates from various methods, we can establish a linear and robust correlation between these two fundamental parameters. Our Hubble diagram reveals a clear linear relationship between redshift and distance, consistent with the idea that the universe is constantly expanding. The slope of this relation is correspondent to the Hubble constant, providing insight into the universe's expansion history. We use the same methods and ideology Hubble did when he announced this breakthrough in 1929. His work has significant importance on the study of our universe and the Big Bang, highlighting the importance of continued efforts to refine our knowledge of the universe. This study will further delve into the reasons behind the decisions to use these values, and to replicate the thought process of one of Physics' greatest minds.

Keywords: Hubble diagram, redshifts, galaxies, relative distance

I. INTRODUCTION

It is known that light emitted by galaxies is shifted towards the red end of the spectrum. This is because they are moving away from us, a phenomenon known as redshift. Consequently, the redshift from the light from galaxies can be used alongside their relative distance to establish a fundamental relationship between the two. Edwin Hubble was the first to do this, and when the relative distance is plotted against the redshift, a linear relationship emerges, giving us the Hubble Diagram. This relationship sparked the conversation of how the universe is constantly expanding and the velocity of the expansion.

In this paper, we describe the results of an observational experiment to determine the relationship between the redshift of light emitted by galaxies and their relative distances from us. This relationship allows us to develop a model for finding the Hubble constant eventually. Specifically, we will utilize a dataset of galaxy observations, where the redshift of light emitted by galaxies is measured spectroscopically and their relative distances are determined through various methods. The Hubble diagram will be constructed by plotting the redshift against the relative distance, allowing us to quantify the expansion rate of the universe.

Our research question is as follows: what is the relationship between the redshift of light emitted by galaxies and their relative distances from us? The redshift is our independent variable, as it is a direct measurement of the galaxy's light. The relative distance is our dependent variable, as it is calculated through various methods.

II. METHODS

A. Finding Distance

The first step in creating a Hubble diagram is finding the distance of a galaxy. Determining the actual or absolute distances is very difficult in astronomy, however, all you need for a Hubble diagram is the relative distances to the galaxies. To measure these distances, we need some way to compare galaxies. One of the easiest ways to do this would be to compare magnitudes. With magnitudes in terms of galaxies, a higher number of magnitude indicates a fainter galaxy, while a lower number refers to a brighter galaxy.

Sloan Digital Sky Survey, or SDSS for short, is a website that houses information on galaxies, and the couple of values we are looking for are the magnitude values. These values are labeled by u, g, r, i, and z, standing for ultraviolet, green, red, and two infrared wavelengths. Once you find a galaxy in the SDSS database, the Object Explorer will show each of these values. For the purpose of this experiment, we will only need one of the five variables, but we must be consistent about which one we use.

Using the magnitude, we can convert this value into a measure of relative distance. To start, we must convert the magnitude from logarithmic units to real units. The magnitude actually measures radiant flux, which is the amount of light that arrives at Earth in a given time. The formula for finding the flux is given as:

$$F = 2.51^{-m} \tag{1}$$

Where F is the radiant flux, and m is the magnitude value. To find the relative distance to the galaxy, we can square root it and inverse it, giving us:

$$d = 1/\sqrt{F} \tag{2}$$

To make this simpler to understand, we can normalize them so the nearest galaxy has a relative distance of 1, and a galaxy twice as far away as the nearest galaxy will have a relative distance of 2. To normalize the relative distances, we can use a ratio between the relative distances of the nearest galaxy and a galaxy you want to find the relative distance of. The equation would be:

$$d_1/d_2 = 1/x \tag{3}$$

Where x would be the normalized distance to galaxy 2, this is what you want to solve for. Repeat this to find the normalized relative distances for the rest of the galaxies.

Another way to estimate the distance to a galaxy, albeit less accurate, is to look at the size of the galaxy on the screen you're looking at it from. Measuring the galaxy in any unit, whether it be inches, centimeters, or something along those lines. The inverse of the measurement will give you the relative distance to the galaxy.

That being said, both of these methods will not be perfect, as different galaxies have different properties. One galaxy may be larger than another, and from Earth, the only information we have would be the image we see. We are not able to tell if the galaxy is larger because it is closer or because it's bigger. To remedy this, we need to look at galaxy clusters.

Galaxy clusters can be thought of as statistical populations of galaxies. The average properties in these clusters should come close to the average properties of galaxies in the universe. This means that we can take a cluster of galaxies and can say that all of the galaxies in the cluster are effectively the same distance from us.

B. Finding Redshift

The next step to creating a Hubble diagram is the redshift. To figure out the redshift of a galaxy, we first need to find the spectrum of a galaxy that shows spectral lines. Then we need to identify which line relates to which atom or molecule. After that, we measure the shift of one of the lines with respect to its wavelength measured on Earth. Finally, we can use a formula to figure out the redshift value.

Let's use an example using a Hydrogen galaxy. Table 1 shows the rest wavelengths of hydrogen on Earth. A spectral line graph on SDSS will give values of alpha, beta, gamma, and delta of hydrogen for the galaxy selected. An example spectral line graph can be seen with Figure 1.

TABLE I. Spectral Lines Information

Name	Color	Wavelength (Angstroms)
Alpha (a)	Red	6562.8
Beta (b)	Blue-green	4861.3
Gamma (g)	Violet	4340.5
Delta (d)	Deep Violet	4101.7

Using the values given by the spectral line graph in Figure 1, we can make a new table with the observed

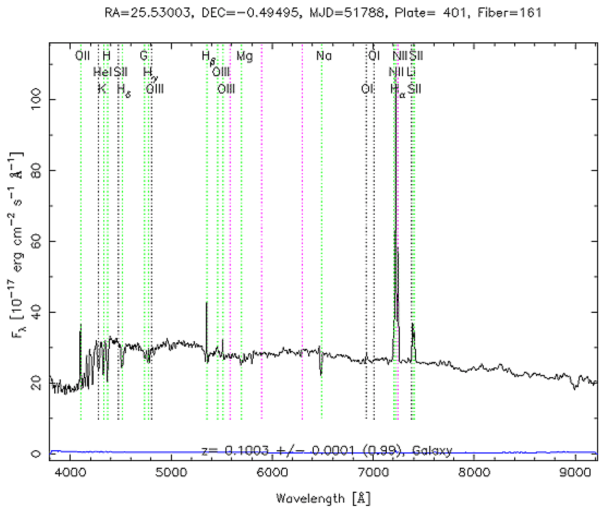


FIG. 1. Spectral line graph

values of wavelength. This is seen in Table 2. Using the values we observed, we can then plug it into equation (4) to find the value of the redshift, z.

TABLE II. Spectral Lines Information

Name	Color	Wavelength (Angstroms)
Alpha (a)	Red	7220
Beta (b)	Blue-green	5360
Gamma (g)	Violet	4780
Delta (d)	Deep Violet	4500

$$1 + z = \lambda_{observed}/\lambda_{rest} \tag{4}$$

The value of z does not change depending on which line you choose, so we can choose whichever we like. Plugging in the numbers and solving, we get a z value of 0.1 for the redshift. If the value of z is negative, we will have a blueshift, or the galaxy is approaching us. With a positive value, the galaxy is moving away from us. Almost every galaxy in the sky has a redshift in its spectrum.

We can convert redshift to velocity if we need to, and we can do that easily with the formula:

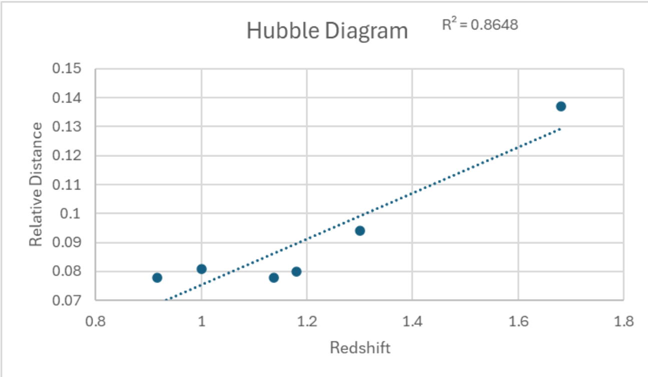
$$v = c * z \tag{5}$$

Where c is the speed of light, and since the formula contains the redshift, we can swap the formula around to:

$$z = v/z \tag{6}$$

Where we can interpret that redshift measures the galaxy's speed of recession relative to the speed of light. The redshift tells us the relative scale of the universe at the time the light left the galaxy. We can use this to write an equation to find the separation distance between our

FIG. 2. Hubble Diagram



galaxy and a specific one, or d(0). This equation can be written:

$$1 + z = d(0)/d(z) \tag{7}$$

Interpreting this formula, if the measured z value is 0.2, that means it corresponds to a time when galaxies were 20 percent closer together than they are now.

III. RESULTS

I went to SDSS to find a galaxy cluster to create a Hubble diagram of. The parameters used for finding the general location are: right ascension: 178.32899, declination: 1.19278. From there, I took note of the magnitude for each of the six galaxies, calculated the relative distance, normalized it, found the redshift from hydrogen, and plotted the graph. Table 3 shows the data acquired, and plotting the data gives us our Hubble diagram.

TABLE III. Hubble Diagram

Magnitude (u)	Relative Distance	Redshift (z)
18.97	1.18016103	0.08
19.74	1.681959959	0.137
18.42	0.916285885	0.078
18.89	1.137507637	0.078
19.18	1.299891221	0.094
18.61	1	0.081

My Hubble diagram has an r squared value of 0.8648, or there is about an 86 percent certainty that there is a linear correlation between distance and redshift. Due to the nature of the experiment, we can conclude that the data does show a linear relationship between distance and redshift.

IV. DISCUSSION

Our research question was as follows: what is the relationship between the redshift of light emitted by galaxies and their relative distances from us? We have found that the relationship is linear, but what does this mean? This breakthrough was key to discovering that the universe is constantly expanding as the linear relationship between distance and redshift does not depend on the direction in the sky, we see redshifts in galaxies wherever we look.

Hubble later discovered that the data in his diagram could be represented by a straight line, or a linear relation. The linear relation between redshift and distance can be written:

$$c * z = H_0 * d \tag{8}$$

Where c is the speed of light, z is the redshift, d is the distance, and H_0 is the Hubble constant, whose value depends on the units used to measure distance. We can safely say that we have validated the formula Hubble discovered long ago, and we can use it to confirm that our universe is expanding.

What I measured here is what Hubble did when this breakthrough was made, however we can more accurately calculate distances with absolute distances. When using an absolute distance for a Hubble diagram, units of megaparsec (Mpc) are used, where 1 Mpc $= 3.1 * 10^{22} m$. Right now, the Hubble constant in Mpc is $H_0 = 70$ km/sec/Mpc.

Even with modern technology, the error of the Hubble constant sits at around 10 percent. This really highlights how uncertain it is to measure absolute distances, but since we only could measure relative distances here, we have no information on the value of H_0. With that being said, Hubble nearly won the Nobel Prize in Physics for doing exactly what I did here.

Though Edwin Hubble had a lot of issues with convincing his fellow scientists of his discovery in 1929. He teamed up with astronomer Milton Humason to create more diagrams for larger distances and higher redshifts. After looking at thousands of galaxies, they succeeded in 1937, almost 8 years after his discovery was announced. By that point, the redshift-distance relation was firmly established.[1]

V. SUMMARY

In summary, we have determined the relationship between distance and the redshift of a galaxy. We figured out how to calculate distances using magnitude, we figured out how to find and calculate redshifts using spectra lines. The relationship of distance and redshift is linear, and is the result of the universe constantly expanding. We used the methods Hubble did back when he first discovered the relation.

[1] Sloan Digital Sky Survey. (n.d.). The Hubble Diagram. https://skyserver.sdss.org/dr1/en/proj/advanced/hubble/default.asp